Dr. Tasneem Hassan, Dr. Alaa Mustafa

One Dimensional Kinematics

3-1 Acceleration

$$\text{Average acceleration} = a_{avg=}\frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\text{Instantaneous acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

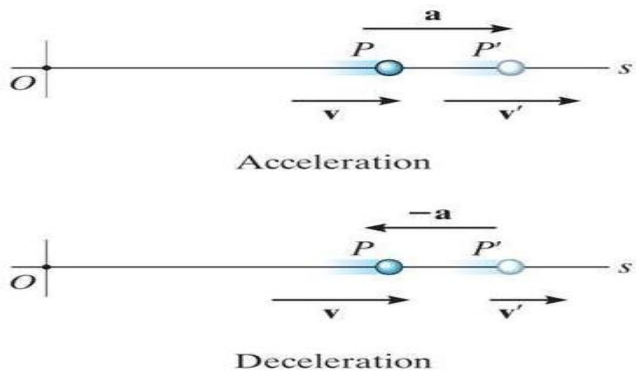
- **Acceleration** is the rate of change in the velocity of a particle.
- The **units** of acceleration are **m/s^2** (length per time squared).
- The **instantaneous acceleration (التعجيل الاني)** is the first time derivative of **velocity v(t)** and the second time derivative of **position x(t)**:

$$a_{avg=}\frac{dv}{dt} = \frac{d_x^2}{d_t^2}$$

- **Vector form: a** = dv / dt

  Scalar form: a = d^2 s / dt^2

- Acceleration can be **positive** (speed increasing) or **negative** (speed decreasing).
- In one-dimensional motion, when the sign on the acceleration and velocity match, the object is said to be "accelerating" (i.e. its speed is increasing). When the signs on acceleration and velocity differ, the object is said to be "decelerating" (i.e. its speed is decreasing)



Example1

A car has ability of accelerating from rest to 160 km/hr in 0.80 seconds. What is the average acceleration of a car?

Solve

$$160\,\frac{km}{hr}\{\frac{1000\,m}{1\,km}\}\{\frac{1\,hr\,1}{60\,min}\}\{\frac{1\,min}{60\,s}\} = 44.444\,\frac{m}{s} \Rightarrow 44\,\frac{m}{s}$$

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v0}{\Delta t} = \frac{44.444 m\,s - 0\,m\,s}{0.80\,s} = 55.555\,\frac{m}{s^2} \Rightarrow 56\,\frac{m}{s^2}$$

3-2 Constant Acceleration

When acceleration is **constant**, the average acceleration and the instantaneous acceleration are the **same** ($a = a_{avg}$).

$$\mathbf{a = a_{avg=}\frac{dv}{dt} = \frac{v - v_0}{t}}$$

Lecture Three: One Dimensional Kinematics

When acceleration is constant, v_{avg}

$$v_{avg} = \frac{(v + v_0)}{2}$$

The **three** kinematic equations can be **integrated** for the special case when acceleration is constant to obtain very useful equations **((In this case, a = g = 9.81 m/s2))**:

1-

$$\frac{dv}{dt} = a = \text{constant} \qquad \int_{v_0}^{v} dv = a\int_{0}^{t} dt \qquad v - v_0 = at$$

$$v = v_0 + at$$

2-

$$\frac{dx}{dt} = v_0 + at \qquad \int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at)dt \qquad x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

3-

$$v\frac{dv}{dx} = a = \text{constant} \qquad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \qquad \tfrac{1}{2}\left(v^2 - v_0^2\right) = a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Example: A particle travels along a straight line to the right with a velocity of v = (4 t – 3 t 2) m/s where t is in seconds. Also, s = 0 when t = 0

<u>Solve</u>

1) Take a derivative of the velocity to determine the acceleration.

$$a = \frac{dv}{dt} = \frac{d}{dt}(4t - 3t^2)$$

$$= 4 - 6t$$

$$when\ t = 4s$$

a= -20 m/s^2

2) Calculate the distance traveled in 4s by integrating the velocity using $s_o = 0$:

$$v = \frac{ds}{dt}$$

$$ds = vdt$$

$$\int_{s_o}^{s} ds = \int_{o}^{t} (4t - 3t^2)dt$$

$s - s_o = 2t^2 - t^3 \Rightarrow s - 0 = 2(4)^2 - (4)^3$

s = – 32 m

2-3 Application of One Dimensional motion with Constant acceleration

Free Fall Bodies

Free-Fall Acceleration An important example of straight-line motion with constant acceleration is that of an object falling freely near Earth's surface. (a = -g =- 9.8 m/s ^{2})

The constant acceleration equations describe this motion, but we make two changes in notation:

(1) We refer the motion to the vertical **y** axis.

(2) We replace **a** with **g**, where g is the magnitude of the free-fall Acceleration.

<u>Equation of motion</u>

$$\mathrm{v} = v_o + gt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\mathrm{y} = \frac{1}{2}(\mathrm{v_o} + \mathrm{v})\mathrm{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\mathrm{y} = v_o t + \frac{1}{2} g t^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$v^2 = v_o^2 + 2gy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Example3

Aston is dropped from rest from the top of a building, after 3s of free fall, what is the displacement y of the stone?

Solve

$$\mathrm{y} = v_o t + \frac{1}{2} g t^2$$

$$\mathrm{y} = 0 + \frac{1}{2}(-9.8)3^2 = -44.1m$$